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Theoretical Study of Water Saturation with Percolation Mechanics Method

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Abstract: Theoretical study of water saturation in oil-water fluid flow area is meaningful to the high, stabilized production and the improvement of ultimate recovery. At present, there is no theoretical description of water saturation in the oil and water seepage process. Combining relational expression between relative permeability and water saturation with the fractional flow equation produces an expression of water cut changed by water saturation; Introducing Taylor expansion and Vieta's theorem to the differentiation of water cut gives a theoretical expression of water saturation in oil-water fluid flow area; Possibility of continuity of the water saturation is also discussed. Applying this formula to certain sandstone reservoir in China achieves good results: In oil-water seepage process, water saturation is continuous in certain interval and the values of it are double in another interval; the oil recovery can be re-calculated and improved with this new equation. This new water saturation equation lays a base for the study of residual oil mobility and enhanced oil recovery.

Keywords: Relative permeability, water saturation, percolation mechanics, Buckley-Leverett equation, Vieta's Theorem

INTRODUCTION

Buckley-Leverett (1942) proposed the Buckley-Leverett equation [1], but we cannot deduce the theoretical expression of water saturation from it. Since then, many scholars began to study the water saturation and already made some progress. The previous study was based on the experimental method, Archie (1942) established a formula of water saturation adaptive to these pure sandstone reservoirs [2]; Poupon's work (1971) [3] was based on Simandoux's which was done in 1983 [4], where an equation of water saturation was given. Fertl (1982) [5] and Dewan (1998) [6] further developed the above-mentioned equation. Yao (1993) et al used the analytical forecasting method to study the water saturation of reservoir [7]; Zhang et al (2008) theoretically deduced the relation between electric resistivity and water saturation [8]; Wang et al (2010) [9] and Li (2010) [10] also studied the water saturation of oil reservoirs with the experimental method, respectively. However, there is seldom theoretical research on water saturation based on the percolation mechanics method. In this paper, we aim to propose a new theoretical expression of water saturation based on percolation mechanics.

ASSUMPTIONS

(1) oil-water fluid flow;

(2) homogeneous porous media;

(3) incompressible rock and liquid;

(4) obey the Darcy's law;

(5) satisfy the 1D Buckley-Leverett equation;

(6) follow the law of conservation of mass.

QUANTITATIVE DESCRIPTION OF WATER SATURATION

A. Quantitative Description of Water Saturation

He, G.S's work (1994) [11] refers

$$\frac{k_o}{k_w} = ae^{-bS_w} \tag{1}$$

By introducing Eq. (1) to the fractional flow equation gives

$$f_w = \frac{1}{1+Mae^{-bS_w}} \tag{2}$$

Where coefficients a, b are determined by properties of rock and fluid and can be solved by graphical method.

The Buckley–Leverett equation is a transport equation used to model two-phase flow in porous media. The Buckley–Leverett equation or the Buckley–Leverett displacement can be interpreted as a way of incorporating the microscopic effects to due capillary pressure in two-phase flow into Darcy's law. In a 1D sample (control volume), let S_w be the water saturation, then the Buckley–Leverett equation is

$$\frac{dx}{dt} = \frac{q(t)}{\phi A}\frac{df_w}{dS_w} \tag{3}$$

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Introducing integration to Eq. (3) gives

$$\frac{\phi A}{W(t)}(x - x_0) = f_w'(S_w) \tag{4}$$

When $x = x_0$ (initial place of oil-water seepage flow area), $S_w = 1 - S_{or}$.

Introducing the differentiation of Eq. (2) to Eq. (4) gives

$$\frac{\phi A}{W(t)}(x - x_0)(1 + Mae^{-bS_w})^2 = Mabe^{-bS_w} \tag{5}$$

By setting $y = e^{-bS_w}$ in Eq. (5) gives

$$\frac{\phi A}{W(t)}(x - x_0)M^2a^2y^2 + (2Ma\frac{\phi A}{W(t)}(x - x_0) - Mab)y + \frac{\phi A}{W(t)}(x - x_0) = 0 \tag{6}$$

Let $\frac{\phi A}{W(t)}(x - x_0)M^2a^2 = A_1, 2Ma\frac{\phi A}{W(t)}(x - x_0) - Mab = A_2, \frac{\phi A}{W(t)}(x - x_0) = A_3$ produces a quadratic equation $A_1y^2 + A_2y + A_3 = 0$.

Solving the quadratic equation and taking the natural logarithm gives

$$S_w = \frac{1}{b}\ln\left(\frac{2\phi A(x - x_0)Ma}{-[2\phi A(x - x_0) - bW(t)] + W(t)\sqrt{b[b - \frac{4\phi A}{W(t)}(x - x_0)]}}\right), 0 < x - x_0 < \frac{bW(t)}{4\phi A} \tag{7}$$

$$S_w = \frac{1}{b}\ln\left(\frac{2\phi A(x - x_0)Ma}{-[2\phi A(x - x_0) - bW(t)] - W(t)\sqrt{b[b - \frac{4\phi A}{W(t)}(x - x_0)]}}\right), 0 < x - x_0 < \frac{bW(t)}{4\phi A} \tag{8}$$

When $x = x_0 + \frac{bW(t)}{4\phi A}$, Eq. (7) and Eq. (8) gives $S_w = \frac{\ln(Ma)}{b}$.

Here, $x_f = x_0 + \frac{bW(t)}{4\phi A}$ is the location of frontal zone in time t. This formula shows that the location of frontal zone goes forward with the increase of oil production, and the water saturation $S_{wf} = \frac{\ln(Ma)}{b}$ should be frontal water saturation.

Introducing the inequality $1 \geq S_w \geq 0$ to Eq. (7) gives $\frac{bW(t)}{2\phi A(Ma + 1)} \leq x - x_0 \leq \frac{bW(t)}{4\phi A}$.

This means the product of total water injected and permeability ratio-saturation slope is less or equal to four times the volume of reservoir that has been flooded. This also refers to the displacement efficiency.

Hence,

$$S_w = \frac{1}{b}\ln\left(2\phi A(x - x_0)Ma \middle/ -[2\phi A(x - x_0) - bW(t)] + W(t)\sqrt{b[b - \frac{4\phi A}{W(t)}(x - x_0)]}\right), \frac{bW(t)}{2\phi A(Ma + 1)} \leq x - x_0 \leq \frac{bW(t)}{4\phi A} \tag{9}$$

Introducing the inequality $1 \geq S_w \geq 0$ to Eq. (8) gives $\frac{bW(t)}{\phi A}\frac{Mae^b}{(Ma + e^b)^2} \leq x - x_0 \leq \frac{bW(t)}{4\phi A}$.

Hence,

$$S_w = \frac{1}{b}\ln\left(2\phi A(x - x_0)Ma \middle/ -[2\phi A(x - x_0) - bW(t)] - W(t)\sqrt{b[b - \frac{4\phi A}{W(t)}(x - x_0)]}\right), \frac{bW(t)}{\phi A}\frac{Mae^b}{(Ma + e^b)^2} \leq x - x_0 \leq \frac{bW(t)}{4\phi A}. \tag{10}$$

B. Possibility of Continuity

The above-mentioned deduction of water saturation shows: the function of water saturation is continuous in interval $\left[\frac{bW(t)}{\phi A}\frac{Mae^b}{(Ma + e^b)^2}, \frac{bW(t)}{2\phi A(Ma + 1)}\right)$; the value of water saturation in interval $\left[\frac{bW(t)}{2\phi A(Ma + 1)}, \frac{bW(t)}{4\phi A}\right)$ is multiple. Here, Possibility of continuity of water saturation function below at $x - x_0$ should be investigated. When x approaches to x_0, f_w approaches to zero, it is, in Eq.(2), we see that b approaches to infinity. So the water saturation value at $x = x_0$ is the value at $x = \frac{bW(t)}{\phi A}\frac{Mae^b}{(Ma + e^b)^2}$.

Note: if there is irreducible water saturation or residual oil saturation, the inequality should be $1 - S_{or} \geq S_w \geq S_{wr}$. And the function of water saturation can be discussed like the above-mentioned process.

C. Improved Oil Recovery

Oil recovery is denoted by E_R, $E_R = \frac{1 - S_w - S_{or}}{1 - S_w}$, so in oil-water seepage flow area, E_R can be calculated with the new expression of water saturation. And through changing certain parameter like ϕ, A, $W(t)$, etc, the value of E_R can also be changed.

APPLICATION AND DISCUSSION

Given the water and oil permeability and practical development data of certain sandstone oil reservoir, see Table **1** and Table **2**.

Table 1. Water and Oil Permeability Data of a Sandstone Oil Reservoir

S_w,%	0	10	20	30	40	50	60	70	75	80	90	100
K_{ro}	1	1	1	0.94	0.80	0.44	0.16	0.045	0	0	0	0
K_{rw}	0	0	0	0	0.04	0.11	0.20	0.30	0.36	0.44	0.68	1

Table 2. Practical Development Data of a Sandstone Oil Reservoir

Velocity ratio of oil and water	porosity	Reservoir width, m	Formation thickness, m	Single well production, m^3/d	residual oil saturation
5	0.25	140	6m	30	0.32

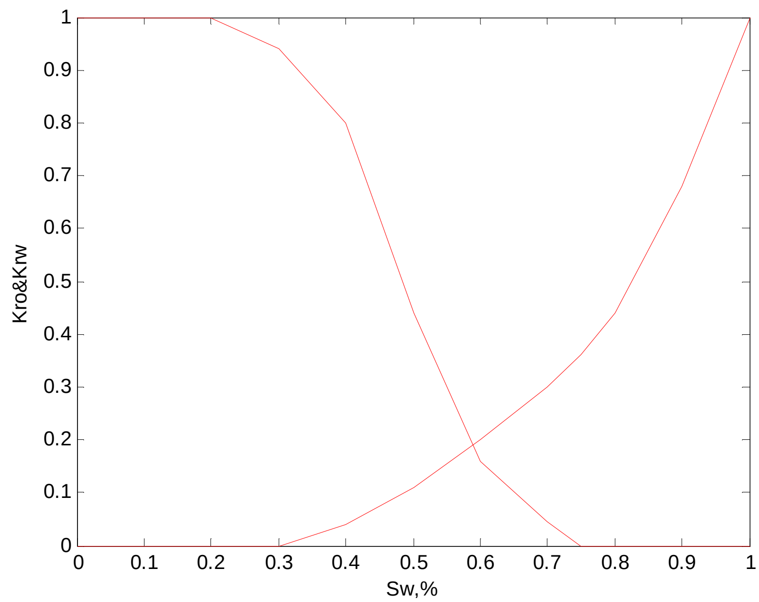

Fig. (1). Broken line graph between effective permeability and saturation

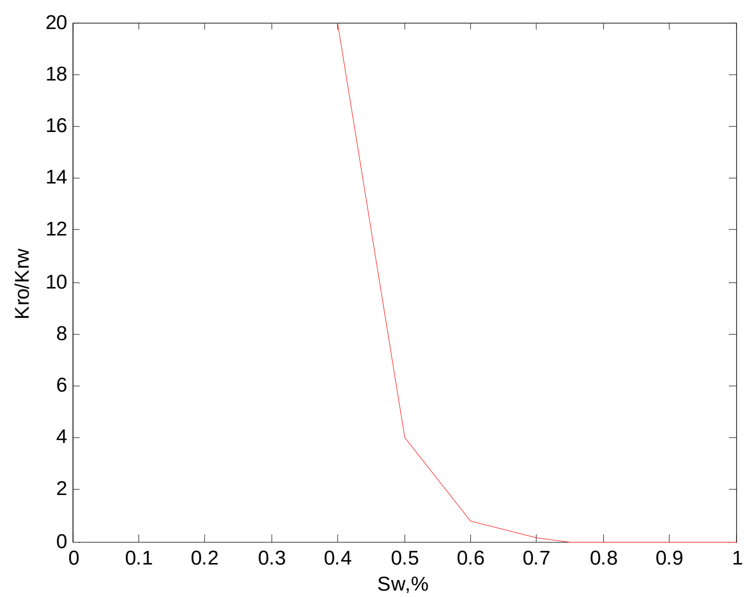

Fig. (2). Broken line graph between effective permeability ratio and water saturation.

From Table **1**, we get the broken line graph between effective permeability and saturation, see Fig. (**1**).

Fig. (**1**). shows that the oleic permeability decreases by the water saturation; the water phase permeability k_w increases by water saturation. From the relation between effective permeability ratio and the given discrete water saturation value, we get the broken line graph between effective permeability ratio and water saturation, see Fig. (**2**).

Fig. (**2**) shows that the effective permeability ratio decreases by water saturation.

By using graphical method gives a=22.39, b=1.52and the theoretical expressions:

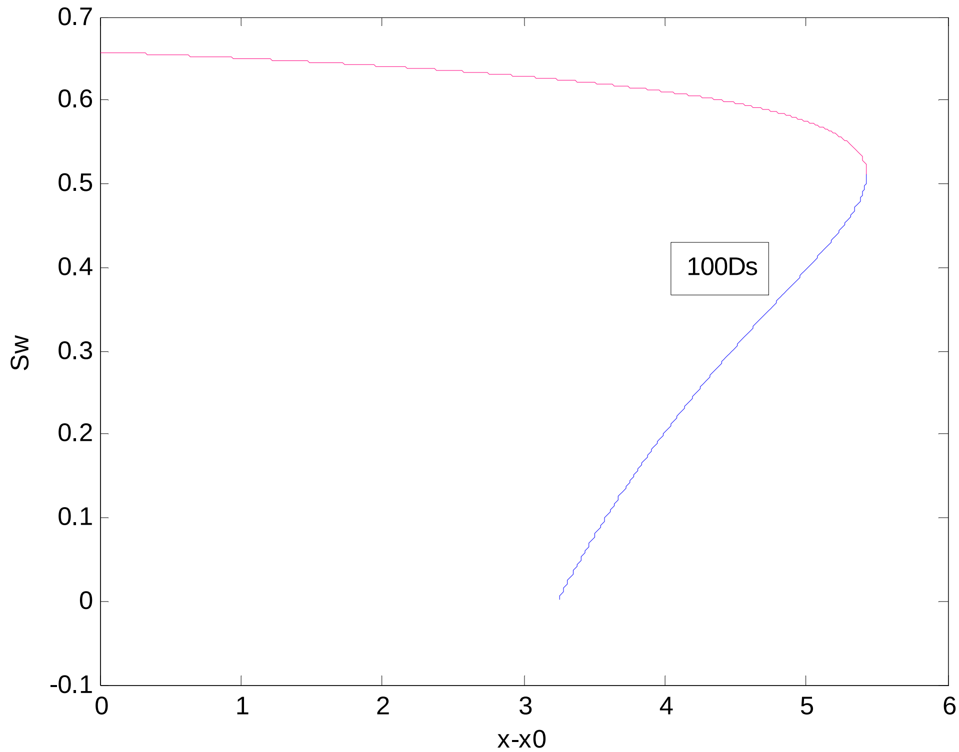

Fig. (3). distribution of water saturation in 100Ds

$$S_w = \frac{1}{1.52} \ln \frac{1880.8(x - x_0)}{45.6t - 420(x - x_0) + 30t\sqrt{2.3104 - \frac{42.56(x - x_0)}{t}}]}, 0.01982t \le x - x_0 \le 0.0543t \quad (11)$$

$$S_w = \frac{1}{1.52} \ln \frac{1880.8(x - x_0)}{45.6t - 420(x - x_0) - 30t\sqrt{2.3104 - \frac{42.56(x - x_0)}{t}}]}, 0.0531t \le x - x_0 \le 0.0543t \quad (12)$$

After fixing the producing time to 100days (Ds) gives Fig. (**3**).

Fig. (**3**). shows that, in oil and water seepage area, the water saturation value is positive and multiple in interval [5.31, 5.43] and the value of water saturation is single in interval [1.982, 5.31] and decreased by time. Here, the oil recovery E_R changes over the water saturation. This new water saturation equation lays a base for the study of residual oil mobility and enhanced oil recovery.

CONCLUSIONS

1. Theoretical expressions of water saturation in oil-water fluid flow area are established;
2. In oil-water seepage process, S_w is continuous in one interval and divergent in another interval;
3. The oil recovery can be re-calculated and improved with this new water saturation function.

NOMENCLATURE

k_o	=	Oleic permeability
k_w	=	Water phase permeability
a	=	Linear intercept
b	=	Linear slope
f_w	=	Water cut or fractional flow rate
M	=	Viscosity ratio of water and oil
ϕ	=	Porosity
A	=	Seepage flow area
S_{or}	=	Residual oil saturation
S_{wr}	=	Irreducible water saturation
x_f	=	Location of frontal zone in time t.
$\partial f_w / \partial S_w$	=	Ratio of change in water cut to change in saturation
$x - x_0$	=	Forwarding distance of any constant water Saturation in seepage area;
$W(t)$	=	Total water injection from the beginning time to t.

CONFLICT OF INTEREST

None declared.

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