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Quantitative Models of Development Laws for Heterogeneous Sandstone Reservoirs by Water Flooding

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Abstract: The Lorenz curve is used to estimate the heterogeneity of an oil reservoir. The corresponding permeability distributions in a series of concept reservoir models with a uniform average permeability but different heterogeneities are obtained and characterized by inverse calculation of the Lorenz curve. Then the development laws of a reservoir in relation to its heterogeneity are studied in method of numerical simulation. The results show that reservoirs with a V_k (permeability variation coefficient is defined in appendix) which characterizes the heterogeneity less than 0.2 have almost the same development rule. It is also found that the permeability profile is consistent with the water-cut profile in both positive and reverse rhythm reservoirs where injected water channels in high permeable layers. Furthermore, the water-cut profile of a reverse rhythm reservoir is more uniform than that of a positive rhythm reservoir. In addition, results also show an exponential relation between water-free recovery and variation coefficients in positive rhythm heterogeneous reservoirs. From the simulation results of reservoir models with different oil-water viscosity ratios, it is found that during the development process, the water-free recovery firstly decreases rapidly with an increasing oil/water viscosity ratio less than 30:1 before it decreases slightly. Besides, a modified logistic function is built to present the relation between ultimate recovery and variation coefficient for positive rhythm reservoirs. The ultimate recovery firstly decreases slightly before the permeability variation coefficient increases up to 0.4, and then decreases sharply. However, the inflexion of variation coefficient in a reverse rhythm one is 0.6. An algebraic equation is also built to describe the water cut change with the recovery factor of recoverable oil reserves. The water-free recovery is higher in a reverse rhythm reservoir than that in a positive rhythm reservoir especially in a reservoir with a relative small oil/water viscosity. However, the ultimate recovery of a reverse rhythm reservoir is higher than that of a positive rhythm reservoir. Also, during development process, the water-cut increases faster in a positive rhythm reservoir than in a reverse rhythm reservoir especially for medium heterogeneous reservoirs.

Keywords: Reservoir heterogeneity, water flooding, lorenz curve, development laws, variation coefficient, quantitative models.

INTRODUCTION

Since the early 1950s, the influence of reservoir heterogeneity on the oil exploitation process has been a concern in petroleum engineering [1]. Reservoir heterogeneity significantly affects both the oil production of a reservoir and the performance of injection wells in water flooding or miscible flooding projects [2, 3]. However, so far few satisfactory techniques are available currently to characterize, quantify and predict the heterogeneity influence due to the complexity of an oil reservoir and the fluids in it [3].

The reservoir heterogeneity is often characterized indirectly by permeability contrast, shale streaks, permeability variation coefficient (interpreted in the Appendix **A**), permeability max-mean ratio (the ratio of maximum permeability to average permeability.), and the extreme permeability ratio (the ratio of maximum permeability to minimum permeability) [4, 5]. The permeability variation coefficient is most widely used among them which can reflect the degree of dispersion of the permeability of a single layer to the average permeability of all layers. Dykstra H method, statistical method and Lorenz curve can be both applied to calculate variation coefficient, however, the first two methods have drawbacks [6, 7].

A series of heterogeneous reservoir models with a uniform average permeability but different values of V_k and rhythms are built to study and quantify the influence of V_k and sedimentary rhythm on water-free recovery, ultimate recovery and water-cut in the development of sandstone heterogeneous reservoirs by water-flooding. This research is of great significance to guide the production schemes in oil field.

1. HETEROGENEOUS RESERVOIR MODEL

Many factors can affect the production performance of a reservoir including the reservoir geological properties, production operation means and well types also [8]. Besides, numerical simulation method is used to investigate the influence of heterogeneity on development characteristics of reservoirs with other properties the same. Besides, the development characteristics of heterogeneous reservoirs with

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specified fluid properties for comparison are also studied by numerical simulation.

1.1. Formation Heterogeneity

Gini factor derived from the Lorenz curve [9] shows the heterogeneity of substances, mostly used in economical evaluation [10]. It is gradually called permeability-variation coefficient when used in the evaluation of the reservoir heterogeneity. Fig. (**1**) shows vertical permeability profiles in a series of concept reservoir models with the same average permeability of $100\times10^{-3}\mu m^2$ but different permeability-variation coefficients. Fig. (**1**) shows that the permeability profile changes more significantly with an increasing permeability-variation coefficient.

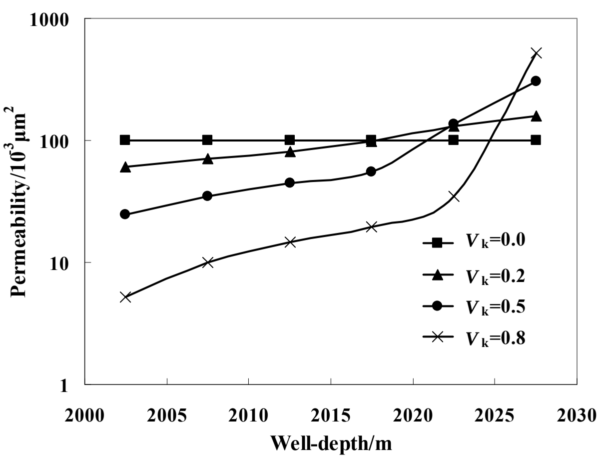


Fig. (1). Permeability distribution along the well bore.

1.2. Model Parameters

Both positive and reverse rhythm reservoir concept models are built; whose depths are all 2000m, oil drainage volume is 150m×150m×30m with a numerical simulation grid system: 30×30×6. These models process the following properties in common: the rock porosity is 0.26, the formation average permeability is $100\times10^{-3}\mu m^2$ the ratio of vertical permeability to horizontal permeability is 0.1, the initial oil saturation is 0.75, the initial water saturation is 0.25, the initial reservoir pressure is 20MPa, the initial formation temperature is 60℃, and the oil viscosity is 20mPa·s at initial temperature. The well pattern is 1/4 of five-spot, and production rate is $2.0m^3/(m\cdot d)$, the relative permeability curve and the capillary pressure curve dependent on the PVT properties are referenced from the Seventh SPE Comparative Solution Project [11].

1.3. Sedimentary Rhythm

Sedimentary rhythm is defined as the regulation that the sand particles deposit in sequence to form rock stratum. It is grouped in positive rhythm, reverse rhythm and composite rhythm. Positive rhythm is composed of several sand layers with the sand diameter declining from bottom to top, i.e. the permeability declines in the vertical profile from bottom to top; reverse rhythm is just reversed (Fig. **2**).

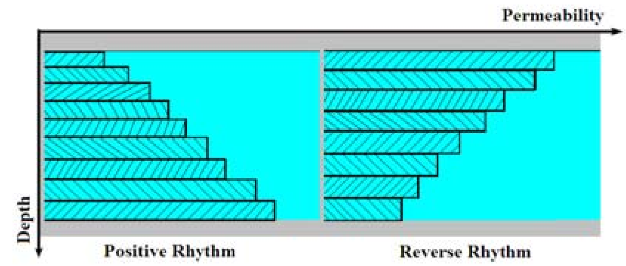


Fig. (2). Sketch map of sedimentary rhythm.

2. CHARACTERIZATION OF WATER-OUT IN HETEROGENEOUS RESERVOIRS

The water-out profile along the well is an exterior reflection of the flow law of oil and water. Among the factors affecting the flow law of oil and water including permeability-variation coefficient, oil-water viscosity ratio, gravity and capillary force and the sedimentary rhythm, the sedimentary rhythm plays the most important role.

2.1. Characterization of Water-out in Positive Rhythm Reservoir

The injected water firstly channels into highly permeable layers having lower resistance and breaks up quickly which leads to a limited volume of injected water and a bad effect of piston-like water drive. Besides, the synergistic effect between layers can also be reduced due to serve heterogeneity among the layers. Based on the parameters labeled in above, a series of models with V_k=0.2, 0.5, 0.8 are built for simulation. The water-cut profiles along the well bore in positive rhythm reservoirs with different extents of formation heterogeneity are plotted in Fig. (**3**) when the oil recovery factors are all equal to 10%. It is obvious in Fig. (**3**) that the water-cut profile changes more significantly in a more heterogeneous reservoir with a larger value of the permeability-variation coefficient. For reservoirs with a certain small permeability-variation coefficient, the water-cut profiles are approximately uniform, and then such kind of reservoirs processing the permeability-variation coefficient smaller than 0.2 can be regarded as homogeneous reservoirs based on the results of in this paper. The water channeling mostly occurs in high permeable layers along the well bore and the dominant water flow in such layers can easily make the water cut reach as high as 95% while the oil recovery is still as low as 10%. In this case, a great proportion of oil will be left in layers with low permeability and some profile control actions need to be taken to enhance oil recovery.

From Fig. (**3**), it is obvious that water-cut profiles (R=10%) are in consistent with the permeability profiles which is because the seepage characteristic of a single layer is basically determined by its distribution to the whole reservoir in terms of permeability. It is also found as both oil and water flow faster in the highly permeable layers, the permeability profile and water-cut profile are in good agreement with each other which has also been approved in field observation [12].

Fig. (**4**) shows the water-cut profiles when oil recovery factor is equal to 10% along the well bore in positive rhythm reservoirs with different oil-water viscosity ratios. From Fig.

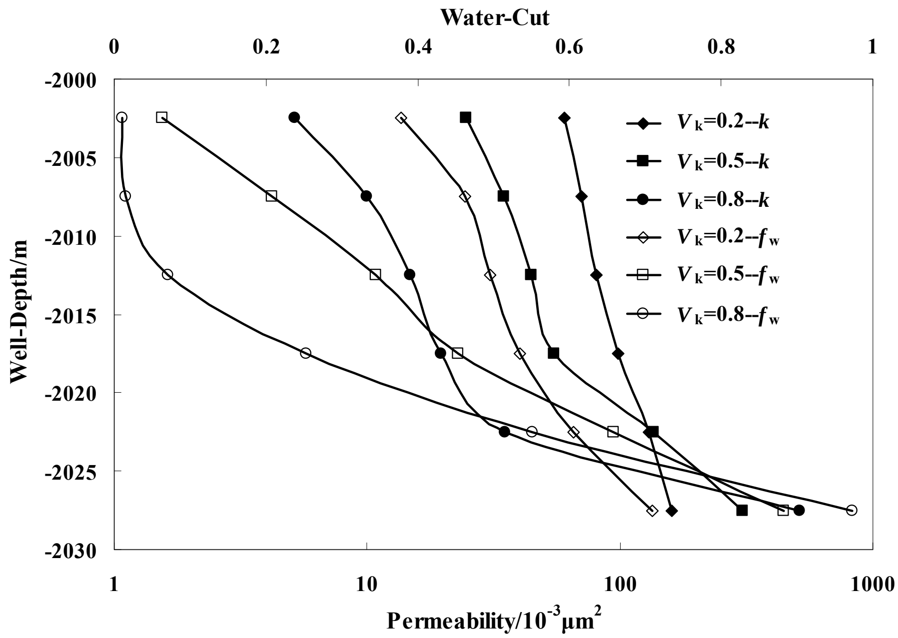


Fig. (3). Permeability & water-cut profiles in positive rhythm reservoirs when *R*=10%.

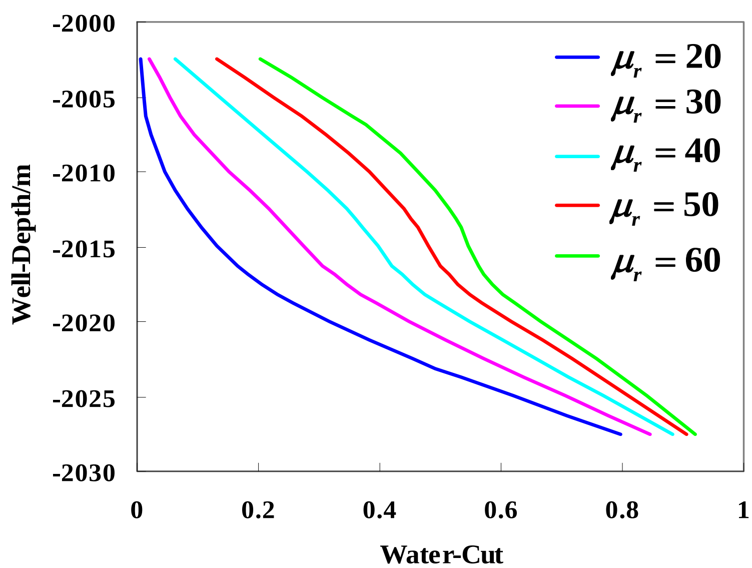


Fig. (4). Water-cut profiles along the well bore of positive rhythm reservoirs with different oil/water viscosity ratios when *R*=10%.

(**4**), it is clear that the water-cut in each layer decreases as the oil-water viscosity ratio reduces, which indicates that the displacement efficiency is higher in reservoirs with smaller oil-water viscosity ratio due to a better effect of a more piston-like water drive. Thus, it is important to improve the mobility control by increasing the viscosity of displacing fluid in the development of reservoirs with higher viscosity oil.

2.2. Characterization of Water-out in Reverse Rhythm Reservoirs

Different from cases in positive rhythm reservoirs, the water-cut profile is affected simultaneously by the driving force, the gravity, and capillary force. In positive rhythm reservoirs, the driving force tends to make the injected water flow into highly permeable layers which lie in the top part of reserve rhythm reservoirs while the gravity tends to make it flow downward the bottom layers. In this case, the water front is more piston-like and a better sweep efficiency can be achieved which will lead to a high oil recovery. Reverse rhythm reservoir concept models with properties labeled above are built in which permeability increases from bottom to top. Fig. (**5**) shows the water-cut profiles and permeability profiles at the same oil recovery *R*=10% along the well bore in reverse rhythm reservoirs with different permeability-variation coefficients. Fig. (**6**) shows the water-cut profiles when *R*=10% along the well bore in reverse rhythm reservoirs with different oil-water viscosity ratios. From Figs. (**5**) and (**6**), it is found that most characteristics of the flow law are similar in both positive and reverse rhythm reservoirs except that the water-cut profiles in reverse rhythm reservoirs are more uniform than that in positive

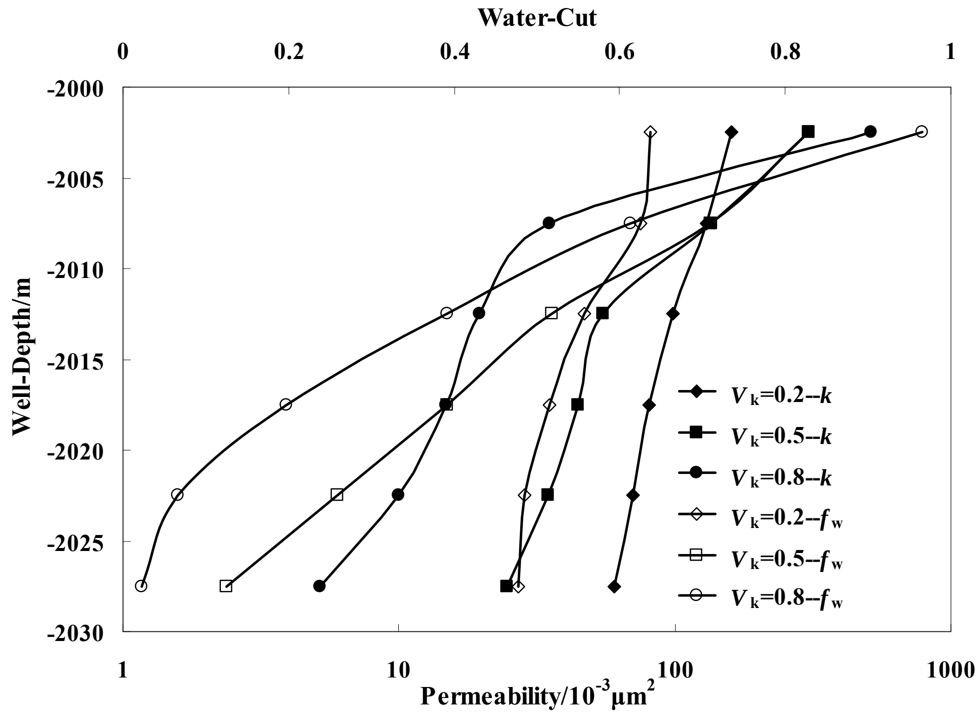


Fig. (5). Permeability & water-cut profiles in reverse rhythm reservoirs when R=10%.

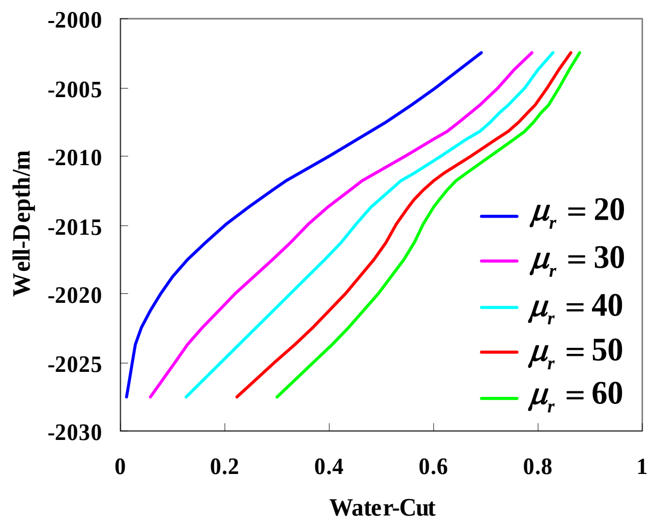


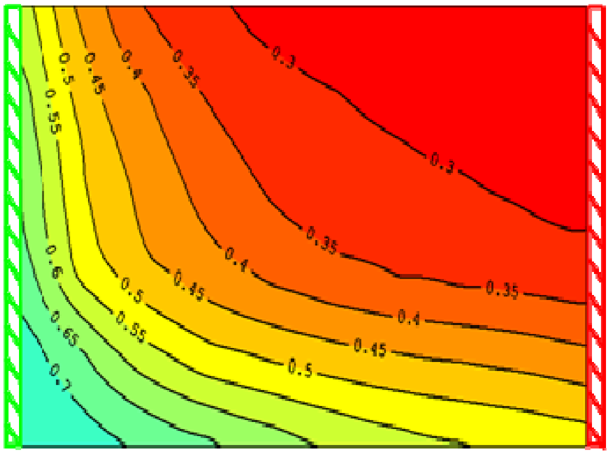
Fig. (6). Water-cut profiles along the well bore of reverse rhythm reservoirs with different oil/water viscosity ratios when R=10%.

rhythm reservoirs due to a synergistic effect of driving force and gravity.

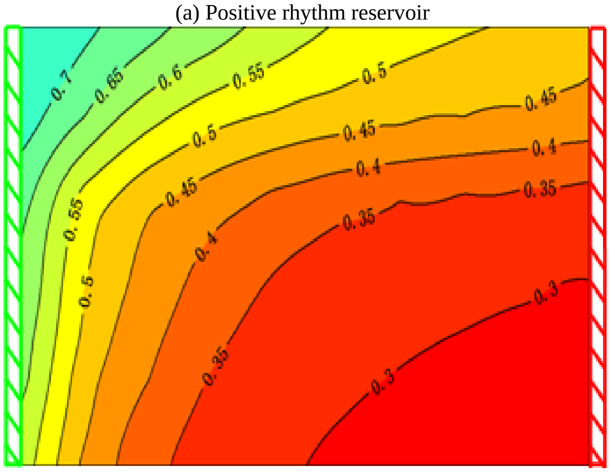
Injected water tends to channel into high permeable layers and this tendency is even stronger with the effect of gravity in positive rhythm reservoirs where permeability increases from top to bottom. Considering gravity effect, the water saturation is distributed in a more uniform manner and much lower in reverse rhythm reservoirs than that in the positive rhythm reservoirs [13]. Fig. (**7**) shows the water saturation profiles both in positive rhythm and reverse rhythm reservoirs with the same V_k=0.5 when the oil recovery R=10%. From Fig. (**7**), it is found that at the top part of reservoirs, the water saturation is higher in positive rhythm reservoirs than in reverse rhythm reservoirs, while lower at bottom part. From the above, it is concluded that the water saturation is more uniform, the swept volume is larger and so the development effect is better in reverse rhythm reservoirs.

3. CHARACTERIZATION OF DEVELOPMENT LAWS FOR HETEROGENEOUS RESERVOIRS

Water-free recovery, ultimate recovery factor and water-cut performance are always highly ranked concerns in oil field production which are deeply affected by main properties of reservoirs including heterogeneity, oil-water viscosity ratio, sedimentary rhythm, relative permeability and so on. Because it is difficult to quantify the influence of relative permeability, a typical relative permeability curve for sandstone reservoirs has been used to broaden the suitability of the theory. In this paper, the influences of heterogeneity and oil-water viscosity ratio on the



(a) Positive rhythm reservoir



(b) Reverse rhythm reservoir

Fig. (7). Formation water saturation profile when R=10% for V_k=0.53.

development performance are studied and quantified in different sedimentary rhythm reservoirs with the parameters used building concept models shown in Table **1**.

3.1. Characteristics of Water-free Recovery Factors in Positive Rhythm Reservoirs

The water-free recovery factor in a reservoir is mainly affected by the permeability-variation coefficient V_k and oil/water viscosity ratio μ_r of the reservoir [12]. Based on the results of numerical simulation, relations between water-free recovery factors and oil/water viscosity ratios in reservoirs with different permeability-variation coefficients are built as shown in Fig. (**8**). From Fig. (**8**), it is found that the water-free recovery factor decreases with the oil/water viscosity ratio and the variation coefficient increasing. First, the water-free recovery decreases rapidly until the oil/water viscosity ratio increases up to 30, and then decreases slightly with no permeability- variation coefficient change. The results also show that, the water-free recovery factor decreases linearly with an increasing value of V_k in a semi-log coordinate system, and so the water-free recovery factor shows an exponential in relation to V_k,

$$\frac{R_f}{R_f|_{V_k=0}} = \exp(\alpha V_k) \tag{1}$$

Where R_f is the water-free recovery, and $R_f|_{V_k=0}$ is the water-free recovery factor for homogeneous formation (V_k=0), which is a function of oil/water viscosity ratio μ_r, shown as below,

$$R_f|_{V_k=0} = 0.65\mu_r^{-0.91} \tag{2}$$

In Equation (1), α is a regression coefficient, and it is also related to oil/water viscosity ratio μ_r as follows,

$$\alpha = -0.013\mu_r - 1.94 \tag{3}$$

From Equation (1) and (3), it is obvious that the coefficient α decreases as the oil/water viscosity ratio increases, and has less influence on water-free recovery when the permeability-variation coefficient is larger.

3.2. Characteristics of Ultimate Recovery Factors in Positive Rhythm Reservoirs

Ultimate recovery is the top concern in oil field production. The main factors affecting the ultimate recovery factor in an oil reservoir include: the heterogeneity, the oil/water mobility ratio and the well pattern. Numerical simulation has been performed to study the influence on the ultimate oil recovery factor of permeability-variation coefficient and the oil/water viscosity ratio in a reservoir with five-spot well pattern. The relations between ultimate recovery factors, different oil/water viscosity ratios and variation coefficients are shown in Fig. (**9**). From Fig. (**9**), it is found that the ultimate oil recovery factor decreases as V_k and oil/water viscosity ratio increase. At a constant water/oil ration, the ultimate recovery decreases slightly when V_k is less than 0.4, and decreases sharply with an increasing V_k larger than 0.4. Besides, oil/water viscosity ratios plays an more important role in the determination of ultimate recovery factors when V_k is small enough while have almost

Table 1. Static Parameters of Numerical Simulation

Depth/m	Pressure/MPa	Temperature/ °C	S_{oi}	S_{wc}
2000	20	60	0.75	0.25
Porosity	μ_o/mPa·s	Dx/m	Dy/m	Dz/m
0.26	5,10,15,20,25,30	5	5	5
NTG	Drainage Volume /m^3	Production Rate/m^3/(d·m)	Permeability/10^{-3} μm^2	V_k
1	150×150×30	2.0	100	0,0.1,…0.8,0.9

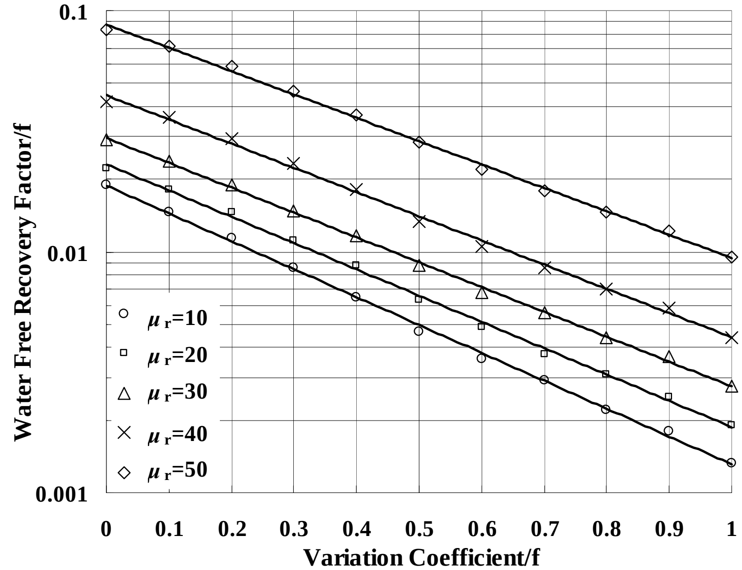


Fig. (8). Water-free recovery characteristics of positive rhythm reservoirs.

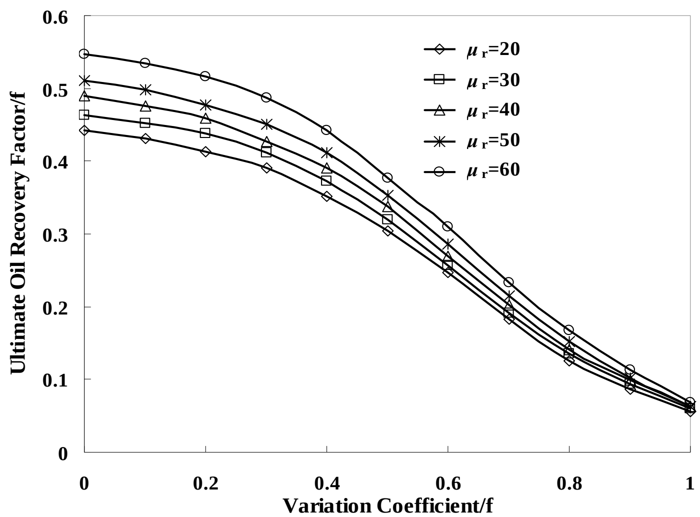


Fig. (9). Ultimate recovery characteristics of positive rhythm reservoirs.

no influence on the ultimate recovery factors when V_k is small enough. A modified logistic function can be used to describe the relation between ultimate recovery factors and variation coefficients in positive rhythm reservoir as follows,

$$\frac{R_E}{R_E|_{V_k=0}} = \frac{1.02}{1+\exp[(V_k - a)/b]} \tag{4}$$

Where R_E is the ultimate recovery, and $R_E|_{V_k=0}$ is the ultimate recovery in homogeneous reservoirs (V_k=0). $R_E|_{V_k=0}$ is a function of oil/water viscosity ratio μ_r, as shown below,

$$R_E|_{V_k=0} = -0.0026\mu_r + 0.595 \tag{5}$$

And *a, b* are constants: *a*=0.64, *b*=0.18.

3.3. Characteristics of Water-cut Performance in Positive Rhythm Reservoirs

The law of water-cut change with oil recovery is a basic law describing the oil production performance [12]. Heterogeneity and mobility ratio are the two most important factors determining how water-cut changes with oil recovery. The water-cut performances are shown in Fig. (**10**) in reservoirs with different variation coefficients but a specified oil/water viscosity ratio of 40:1. From Fig. (**10**), it is found that water-cut increases faster and the ultimate recovery factor decreases in a reservoir with a larger V_k. In addition, the rates of water-cut increase are almost the same in reservoirs with V_k=0.0 and V_k=0.2, which also proves the conclusions that reservoirs with a V_k less than can be regarded as homogeneous reservoirs. By using the method of data fitting, the relation between the water-cut and the oil recovery factor of recoverable reserves is achieved as below:

$$f_w = \left\{1 - \frac{1-R^*}{\left[1+m\cdot\left(R^*\right)^n\right]}\right\} \times f_{EL} \tag{6}$$

Where R^* is the oil recovery factor of recoverable reserves, $R^*=R/R_E$; and f_{EL} is the economy limit water-cut which is set to be 0.98 in this paper; and *m*, *n* are regression coefficients, which are related to permeability-variation coefficient V_k and oil-water viscosity ratio, as shown in Fig. (**11**) and Equation (7),

$$n = -0.778\ln(\mu_r) + 4.05 \quad (7)$$

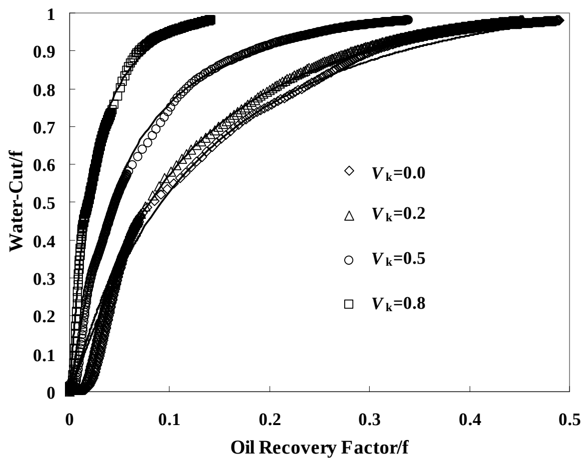


Fig. (10). Water-cut performance characteristics of positive rhythm reservoirs.

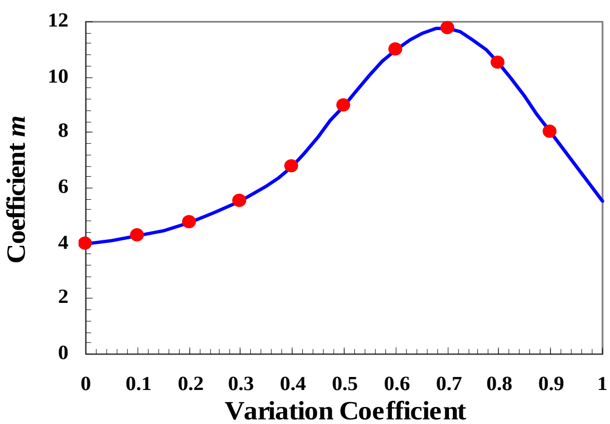


Fig. (11). Coefficients *n* of the model.

3.4. Application of These Models

A multizone positive rhythm sandstone reservoir containing three sand beds of X oil field which has been developed by water-flooding for about 20 years in China is chosen to validate the development laws found in this paper. The properties of the reservoir are shown in Tables **2** and **3**.

With the oil viscosity 28.95mPa·s and V_k=0.76, the water-free recovery, the ultimate recovery and the water-cut performances of the reservoir can be calculated by Equation (1) - Equation (7). From the results shown in Table **4** and Fig. (**12**), it is clearly fund that the simulated results are fairly acceptable.

3.5. Comparison of Development Characteristics for Different Sedimentary Rhythm Reservoirs

In order to study the effect of sedimentary rhythm, different reservoir models with the same static parameters but different sedimentary rhythm are built for simulation. The simulation results shows a better development effect in reverse rhythm reservoirs than in positive rhythm reservoirs due to a synergetic effect of driving force and gravitational differentiation. Fig. (**13a**) shows the comparison of water-free recovery in different sedimentary rhythm reservoirs. The water-free recovery factors in reverse rhythm reservoirs are larger than that in positive rhythm reservoirs especial when the oil/water viscosity ratio is small. Fig. (**13b**) shows the comparison of ultimate recovery factors in different sedimentary rhythm reservoirs, the ultimate recovery factors in reverse rhythm formation are larger. Besides, the ultimate recovery declines slowly in reverse rhythm reservoirs with an increasing V_k less than 0.6 while in positive rhythm reservoirs, the ultimate recovery factors begin to decrease sharply with an increasing V_k once V_k is larger than 0.4. In addition, the ultimate oil recovery factors differ more when the oil/water viscosity ratio is low. And the corresponding V_k for the maximum difference will decrease as oil/water viscosity ratio increases. Fig. (**13c**) shows the comparison of water-cut performance in different sedimentary rhythm reservoirs, the water-cut in reverse rhythm formation is always lower, especially in reservoirs with a medium

Table 2. Properties of Each Sand Bed

Sand Beds	Average Thickness/m	Porosity/%		Permeability($10^{-3}\mu m^2$)			Drilling Ratio
		Span	Mean	Span	Mean	V_k	
S_1	11.28	6.85-16.3	20.456	0.93-1920.21	112.95	0.7596	1.00
S_2	10.08	5.5-17.2	19.454	0.75-439.59	49.17	0.8324	0.89
S_3	7.44	8.35-15.63	24.65	0.45-287.4	32.73	0.6877	0.73

Table 3. Properties of This Reservoir

Depth/m	Viscosity /mPa·s	Pressure /MPa	Temperature /℃	S_{oi}	Well-Pattern	Well Spacing /m	Pay Thickness /m
2400	5.2-52.7	28.2	75	0.72	five-spot	300	22.5

Table 4. Comparison of Production Data of Oil Field and Calculation by Models

Items	Real Value/%	Calculation Value/%	Error/%
Water-free Recovery	0.208	0.192	7.692
Ultimate Recovery	15.06	14.206	5.671

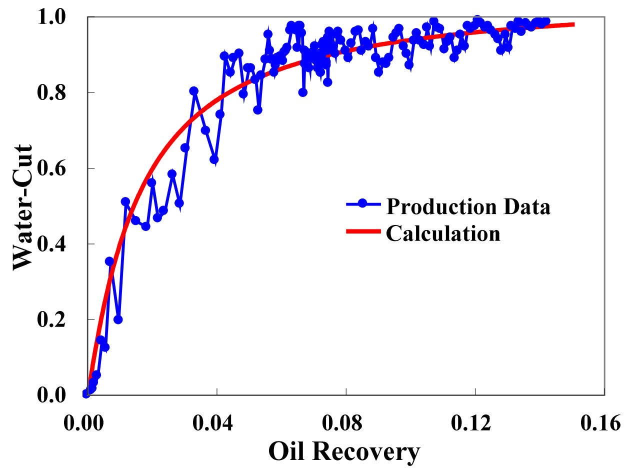
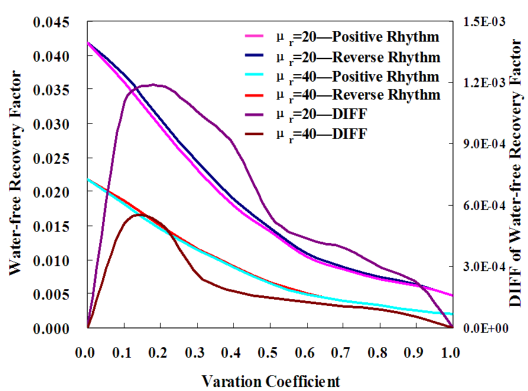
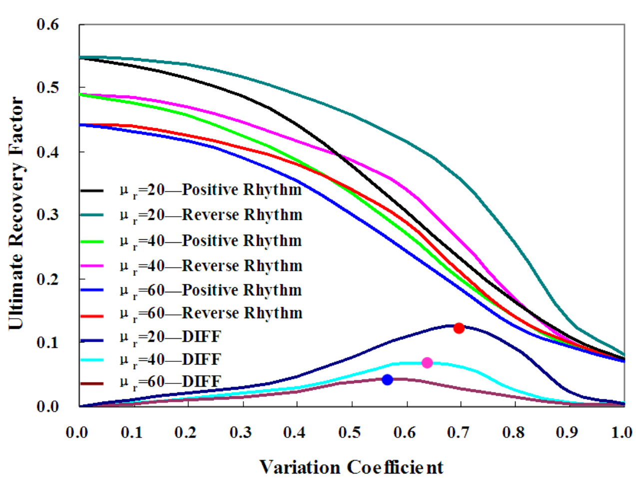


Fig. (12). Comparison of production data of oil field and calculation by quantitative models heterogeneity.

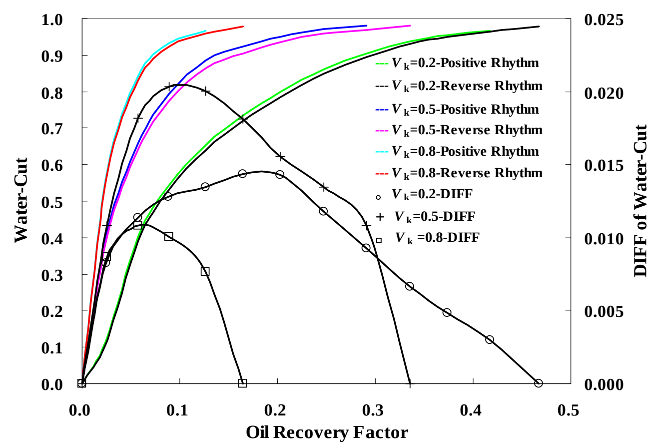


(a) Water-free recovery

b.



(**b**) Ultimate recovery



(**c**) Water-cut performance

Fig. (13). Comparison of development characteristics for different sedimentary rhythm reservoirs.

DISCUSSION

1. With the simulation results of a series of positive rhythm heterogeneous reservoirs, the relations between the water-free oil recovery, the ultimate oil recovery and the water-cut are quantified. However, these relations can be valid only for the reservoirs with a good property of homogeneity in plane since models used in this study are only vertically heterogeneous.
2. The objects of this study are conditional sandstone reservoirs containing conditional light oil and so the applications of the relations would be limited in heavy oil reservoirs.

3. The relations obtained in this paper are validated by a realistic water-flooding example in a conditional sandstone reservoir. Since this example is selected in random in the light oil sandstone reservoirs with a good property of homogeneity in plane, it is proper to apply such relations in other reservoirs with similar properties.

CONCLUSIONS

1. The permeability and water-cut profiles are in a good agreement with each other in both positive and reverse rhythm formations, though the permeability and water-cut profiles are more uniform in reverse rhythm reservoirs. Besides, the swept volume is larger leading to a better development effect in reverse rhythm formation due to a synergetic effect of driving force and gravitational differentiation.
2. The water channeling occurs more easily in reservoirs with a more seriously heterogeneous property and a higher oil-water viscosity ratio, and the lowers water-free recovery is. The water-free recovery decreases rapidly as the oil/water viscosity ratio increases up to 30, and then decreases slightly. However, the water-cut decreases linearly with V_k increasing in a smi-log coordinate system. In addition, the Water-free recovery shows an exponential relation with variation coefficient in positive rhythm reservoirs with the relation coefficient being a function of oil-water viscosity ratio.
3. The ultimate oil recovery factor decreases with a V_k and water-oil ratio increasing. Besides, the ultimate recovery decreases slightly when V_k is less than 0.4, and then decreases sharply. The oil/water viscosity ratio plays more important role in the ultimate recovery factor than V_k does when V_k is low. However, V_k becomes a dominant factor affecting the ultimate oil recovery when V_k is large enough. A modified logistic function is built to show the relation between ultimate recovery and variation coefficient in positive rhythm reservoirs.
4. Reservoirs with the permeability-variation coefficient less than 0.2 can be considered as homogeneous reservoirs. The water-cut change with oil recovery factor is quantified by a regression formula considering the influence of permeability variation coefficient and oil/water viscosity ratio since the coefficient *m* changes with variation coefficient and *n* with oil/water viscosity ratio in this formula.
5. These quantitative characterizations of development laws in heterogeneous reservoirs are validated by a real example in an oil field by water flooding in China.
6. The water-free recovery factors in reverse rhythm reservoirs are larger than that in positive rhythm reservoirs with same oil/water viscosity ratio and permeability variation coefficient especially when the oil/water viscosity ratio is small. The ultimate recovery factors in reverse rhythm reservoirs are always larger than that in positive reservoirs with the same other properties. In addition, the ultimate oil recovery factors differ more when the oil/water viscosity ratio is low. And the corresponding V_k for the maximum difference will decrease as oil/water viscosity ratio increases. The water-cut in reverse rhythm formation is always lower than that in positive rhythm formation especial when V_k is in a medium range.

APPENDIX A-LORENZ CURVE

The Lorenz Curve (Fig. **A1**(**a**)) developed by Max O. Lorenz, an American statistician, was initially used to represent income distribution, where it shows for the bottom *x*% of households, what percentage *y*% of the total income they have. The percentage of households is plotted on the *x*-axis, the percentage of income on the *y*-axis. As shown in Fig. **A1**(**a**), the diagonal AC with an angle of 45 degree indicates a perfect homogeneous reservoir and the broken line ADC indicates a perfect heterogeneous reservoir which means the permeability of the reservoir is centralized in a single point in space. Usually, the Lorenz curves are up-convex curves

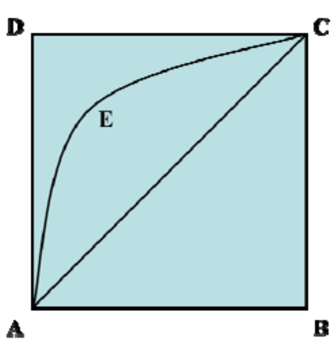


Fig. A1(a) Lorenz Curve

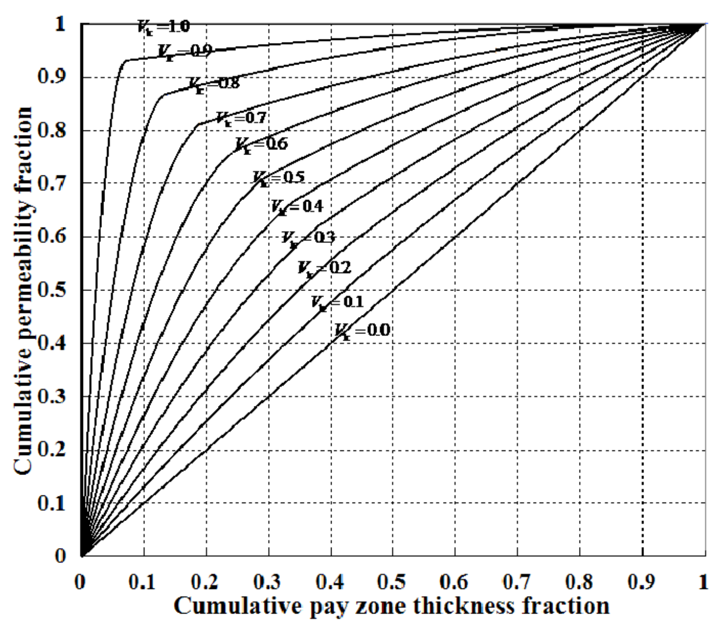


Fig. (A1). (**a**) Lorenz Curve**.** (**b**) Lorenz Curve of different permeability variations.

like line AEC (Fig. **A1**(**a**)), located in between the two typical lines. And the Gini factor is used to quantify the extent of heterogeneity as below:

$$G = \frac{S_{\text{ACEA}}}{S_{\text{ACDA}}} = \frac{\int_0^1 y dx - \frac{1}{2}}{\frac{1}{2}} = 2\int_0^1 y dx - 1 \quad \text{(A1)}$$

In recent years, the Lorenz Curve is often used to characterize the heterogeneity of a reservoir in a field-scale in petroleum engineering as an analogy used in statistics in a method described as follows: Fist, permeability values (k_i) of n different layers are ranked from high to low, and also determine the thickness (h_i) of each corresponding layer; second, after we get n sets of (k_i,h_i), we calculate a series of sets of (x_m, y_m) to plot the Lorenz Curve with abscissa representing the cumulative pay zone thickness percentage x_m and ordinate representing the cumulative permeability percentage y_m(Fig. **A1**(**b**)).

$$x_m = \frac{\sum_{i=1}^{m} h_i}{\sum_{i=1}^{n} h_i} - y_m = \frac{\sum_{i=1}^{m} k_i h_i}{\sum_{i=1}^{n} k_i h_i} \quad m \le n \quad \text{(A2)}$$

Each point in the Lorenz curve physically shows the percentage of the cumulative permeability of the cumulative pay zone thickness at this point. Similar with the definition of Gini coefficient, V_k is defined as the coefficient of variation regarding the heterogeneity of a reservoir:

$$V_k = 2\int_0^1 \frac{\sum_{i=1}^{m} k_i h_i}{\sum_{i=1}^{n} k_i h_i} d\left(\frac{\sum_{i=1}^{m} h_i}{\sum_{i=1}^{n} h_i}\right) - 1 \quad \text{(A3)}$$

In order to quantify the influence of heterogeneity on development performance in a reservoir, a series of Lorenz curves corresponding to different values of V_k (Fig. **A1**(**b**)) are generated based on the definition of V_k to get different permeability distributions in all layers with the same average permeability . The method to determine the permeability of each layer at a specified V_k is described in detail as below:

1. Generate a series of Lorenz Curves corresponding to different values of V_k; (Fig. **A1**(**b**)) V_k=0.0……1.0)
2. Determine the number of layers and the thickness of each layer; (In this paper, n is simplified to be 6 and h_i to be 5 m)
3. With the Loren Curves got above, we get a series of sets of (x_1,y_1), (x_2,y_2), (x_3,y_3), (x_4,y_4), (x_5,y_5), (x_6,y_6);
4. From Equation (A3), values of permeability (k_1,…, k_6) for each layer can be calculated.

CONFLICT OF INTEREST

None declared.

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