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Modified Theoretical Expression of Water Saturation in Oil-Water Fluid Flow Area

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Abstract: Theoretical study of water saturation in oil-water fluid flow area is meaningful to the high, stabilized production and the improvement of ultimate recovery. Combining relational expression between relative permeability and water saturation with the fractional flow equation produced an expression of water cut changed by water saturation; Introducing Vieta's theorem to the differentiation of water cut gave a theoretical expression of water saturation in oil-water fluid flow area; Introducing the concept of frontal water saturation to these expressions give the continuous expression of water saturation. Applying this formula to certain sandstone reservoir in China achieves good results: In oil-water seepage process, water saturation is increasing by frontal distance, and the frontal distance increases over time. This new water saturation equation lays a base for the study of residual oil mobility and enhanced oil recovery.

Keywords: Relative permeability, percolation mechanics, Buckley-Leverett equation, Vieta's Theorem, frontal water saturation.

1. INTRODUCTION

Buckley-Leverett (1942) proposed the Buckley-Leverett equation [1], but we cannot deduce the theoretical expression of water saturation from it. Since then, many scholars began to study the water saturation and already made some progress. The previous study was based on the experimental method, Archie (1942) established a formula of water saturation adaptive to these pure sandstone reservoirs [2]; Poupon's work (1971)[3] was based on Simandoux's which was done in 1983 [4], where an equation of water saturation was given. Fertl (1982) [5] and Dewan (1998) [6] further developed the above-mentioned equation. Yao (1993) *et al.* used the analytical forecasting method to study the water saturation of reservoir [7]; Zhang *et al.* (2008) theoretically deduced the relation between electric resistivity and water saturation [8]; Wang *et al.* (2010) [9] and Li (2010) [10] also studied the water saturation of oil reservoirs with the experimental method, respectively. However, there is seldom theoretical research on water saturation based on the percolation mechanics method. In this paper, we aim to propose a new theoretical expression of water saturation based on percolation mechanics.

2. ASSUMPTIONS

(1) Oil-water fluid flow;
(2) Homogeneous porous media;
(3) Incompressible rock and liquid;
(4) Obey the Darcy's law;
(5) Satisfy the 1D Buckley-Leverett equation;
(6) Follow the law of conservation of mass;

3. QUANTITATIVE DESCRIPTION OF WATER SATURATION

Quantitative description of water saturation

He, G.S's work (1994) [11] refers

$$\frac{k_o}{k_w} = ae^{-bS_w} \tag{1}$$

By introducing Eq. (1) to the fractional flow equation gives

$$f_w = \frac{1}{1 + Mae^{-bS_w}} \tag{2}$$

Where coefficients a, b are determined by properties of rock and fluid and can be solved by graphical method.

The Buckley–Leverett equation is a transport equation used to model two-phase flow in porous media. The Buckley–Leverett equation or the Buckley–Leverett displacement can be interpreted as a way of incorporating the microscopic effects to due capillary pressure in two-phase flow into Darcy's law. In a 1D sample (control volume), let S_w be the water saturation, then the Buckley–Leverett equation is

$$\frac{dx}{dt} = \frac{q(t)}{\phi A}\frac{df_w}{dS_w} \tag{3}$$

Introducing integration to Eq. (3) gives

$$\frac{\phi A}{W(t)}(x - x_0) = f_w'(S_w) \tag{4}$$

When $x = x_0$ (initial place of oil-water seepage flow area), $S_w = 1 - S_{or}$.

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Introducing the differentiation of Eq. (2) to Eq. (4) gives

$$\frac{\phi A}{W(t)}(x-x_0)(1+Mae^{-bS_w})^2 = Mabe^{-bS_w} \quad (5)$$

By setting $y = e^{-bS_w}$ in Eq. (5) gives

$$\frac{\phi A}{W(t)}(x-x_0)M^2a^2y^2 + (2Ma\frac{\phi A}{W(t)}(x-x_0) - Mab)y + \frac{\phi A}{W(t)}(x-x_0) = 0 \quad (6)$$

Let

$$\frac{\phi A}{W(t)}(x-x_0)M^2a^2 = A_1, 2Ma\frac{\phi A}{W(t)}(x-x_0) - Mab = A_2, \frac{\phi A}{W(t)}(x-x_0) = A_3$$

produces a quadratic equation $A_1y^2 + A_2y + A_3 = 0$.

Solving the quadratic equation and taking the natural logarithm gives [12]

$$S_w = \frac{1}{b}\ln\left(\frac{2\phi A(x-x_0)Ma}{-[2\phi A(x-x_0) - bW(t)] + W(t)\sqrt{b[b - \frac{4\phi A}{W(t)}(x-x_0)]}}\right), 0 < x - x_0 < \frac{bW(t)}{4\phi A} \quad (7)$$

$$S_w = \frac{1}{b}\ln\left(\frac{2\phi A(x-x_0)Ma}{-[2\phi A(x-x_0) - bW(t)] - W(t)\sqrt{b[b - \frac{4\phi A}{W(t)}(x-x_0)]}}\right), 0 < x - x_0 < \frac{bW(t)}{4\phi A} \quad (8)$$

When $x = x_0 + \frac{bW(t)}{4\phi A}$, Eq. (7) and Eq. (8) gives $S_w = \frac{\ln(Ma)}{b}$.

Here, $x_f = x_0 + \frac{bW(t)}{4\phi A}$ is the location of frontal zone in time t. This formula shows that the location of frontal zone goes forward with the increase of oil production, and the water saturation $S_{wf} = \frac{\ln(Ma)}{b}$ should be frontal water saturation.

Introducing the inequality $S_{wf} \geq S_w \geq 0$ (instead of $1 \geq S_w \geq 0$) to Eq. (7) gives $\frac{bW(t)}{2\phi A(Ma+1)} \leq x - x_0 \leq \frac{bW(t)}{4\phi A}$.

Hence, the distribution equation of water saturation becomes:

$$S_w = \frac{1}{b}\ln\left(2\phi A(x-x_0)Ma \Big/ -[2\phi A(x-x_0) - bW(t)] + W(t)\sqrt{b[b - \frac{4\phi A}{W(t)}(x-x_0)]}\right), \frac{bW(t)}{2\phi A(Ma+1)} \leq x - x_0 \leq \frac{bW(t)}{4\phi A} \quad (9)$$

Introducing the inequality $S_{wf} \geq S_w \geq 0$ (instead of $1 \geq S_w \geq 0$) to Eq. (8) gives no solution, hence the distribution equation of water saturation is eq. (9). Here, the inequality $S_{wf} \geq S_w \geq 0$ is the guarantee of single value of water saturation.

Note: the above-mentioned deduction of water saturation shows: the function of water saturation is continuous in interval $\left[\frac{bW(t)}{2\phi A(Ma+1)}, \frac{bW(t)}{4\phi A}\right]$, and this water saturation distribution equation is adaptive to any water driving oil reservoir.

4. APPLICATION AND DISCUSSION

Given the water and oil permeability and practical development data of certain sandstone oil reservoir, see (Table **1** and Table **2**).

By using graphical method gives $a = 22.39$, $b = 1.52$ and the theoretical expressions:

$$S_w = \frac{1}{1.52}\ln\frac{1880.8(x-x_0)}{45.6t - 420(x-x_0) + 30t\sqrt{2.3104 - \frac{42.56(x-x_0)}{t}}]}, 0.01982t \leq x - x_0 \leq 0.0543t \quad (10)$$

After fixing the producing time to 100, 200, 300, 400, 500, 600days (Ds) gives (Fig. **1**).

Fig. (**1**) shows that, in oil and water seepage area, water saturation is increasing by frontal distance, and the frontal distance increases over time. When fixing the producing time from 100 days to 600 days, the distribution curve water saturation changed after producing for 500 days, compared to the 400 days producing time.

5. CONCLUSION

1. Modified theoretical expression of water saturation based on reference [12] in oil-water fluid flow area is established.
2. In oil-water seepage process, water saturation is increasing by frontal distance, and the frontal distance increases over time.

Table 1. Water and Oil Permeability Data of a Sandstone Oil Reservoir

S_w,%	0	10	20	30	40	50	60	70	75	80	90	100
K_{ro}	1	1	1	0.94	0.80	0.44	0.16	0.045	0	0	0	0
K_{rw}	0	0	0	0	0.04	0.11	0.20	0.30	0.36	0.44	0.68	1

Table 2. Practical Development Data of a Sandstone Oil Reservoir

Velocity Ratio of Oil and Water	Porosity	Reservoir Width, m	Formation Thickness, m	Single Well Production, m^3/d	Residual Oil Saturation
5	0.25	140	6m	30	0.32

Fig. (1). Distribution of water saturation in 100Ds.

CONFLICT OF INTEREST

The author(s) confirm that this article content has no conflicts of interest.

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NOMENCLATURE

k_o = Oleic permeability

k_w = Water phase permeability

a = Linear intercept

b = Linear slope

f_w = Water cut or fractional flow rate

M = Viscosity ratio of water and oil

ϕ = Porosity

A = Seepage flow area

S_{or} = Residual oil saturation

S_{wr} = Irreducible water saturation

x_f = Location of frontal zone in time t

$\partial f_w / \partial S_w$ = Ratio of change in water cut to change in saturation

$x - x_0$ = Forwarding distance of any constant water saturation in seepage area

$W(t)$ = Total water injection from the beginning time to t

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