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Productivity Model of Multi-Layer Reservoir with Blocks

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Abstract: Based on the assumption of multi-layer reservoir with complex blocks and combined with the wellbore flow equations, we established a percolation mathematical model for the coupled multi-layer reservoir with blocks and wellbore, and then we utilized the Laplace change, Stehfest inversion and conjugate gradient method, *etc.* to solve the model. Then, we took the multi-layer reservoir with complex blocks as an example to simplify the shape of the reservoir and calculate the distribution of pressure and production of the horizontal well when it passes through two sand bodies. Finally, we calculated the dimensionless pressure derivative and productivity of four different types of oil wells (or have different relative positions). This model has provided a theoretical basis to select the reasonable well types for the multi-layer reservoir with complex blocks.

Keywords: Fault block, horizontal well, mathematical model, multi-layer, productivity, well type.

1. INTRODUCTION

The efficient development of multi-layer reservoir with complex blocks and low well control is a technical difficulty faced by many researchers. It requires vertical wells, horizontal wells, highly deviated wells and cluster wells and other complex well types, so it is necessary to select a reasonable and effective type.

Due to the complexity of the reservoir, the well structure and its flow characteristics, the fluid flow in the wellbore becomes very complicated, which has an impact on the productivity design production, selection of the well types and the deployment of well patterns [1]. Therefore, nowadays it is still necessary to draw the productivity design of vertical and horizontal wells. It has been found that this kind of productivity design needs to be further optimized.

Ouyang *et al.* established the single-phase flow pressure gradient model of the wellbore [2, 3], and the combined model of wellbore flow of one or several cluster wells in an infinite reservoir and the reservoir flow [4]. Gringarten and Ramey established the source function of closed reservoir flow equations, which has laid the theoretical foundation for the percolation model [5]. Therefore, we have established a percolation model with one or several oil and water wells in a closed reservoir; each well contains several branch wells which contain vertical, inclined and horizontal sections. Each well is contained in a closed reservoir or passes through multiple disconnected oil layers.

2. ESTABLISHMENT OF PRODUCTIVITY EQUATION

2.1. Reservoir Flow Equations

Assume that an oil well passes through several disconnected rectangle oil sands in the vertical direction, and it may be a vertical well, inclined well or a combination of both.

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(1) All the layers have the same or similar initial flow potential.

(2) All the layers are in the same pressure system and the pressure transfer among various oil layers is instantly completed by the wellbore.

(3) The homogeneity of the reservoir is aeolotropic. However, it has different porosity, permeability and integrated compression factors whose nature does not change with the variation of pressure.

(4) The border of oil layers are closed border.

(5) The formation fluid is the single-phase compressible fluid, and the fluid of different layers has different compressibility and viscosity.

The flow equation of the jth layer can be described as:

$$Kx_j \frac{\partial^2 \varphi}{\partial x_j^2} + Ky_j \frac{\partial^2 \varphi}{\partial y_j^2} + Kz_j \frac{\partial^2 \varphi}{\partial z_j^2} = \phi_j \mu_j C_{tj} \frac{\partial \phi}{\partial t} \tag{1}$$

To give the equation (1) a general solution, conduct dimensionless treatment.

Dimensionless fluid potential φ_{Dj} is:

$$\varphi_{Dj} = \frac{\varphi_i - \varphi\left(x_j, y_j, z_j, t\right)}{\varphi_i} \tag{2}$$

Dimensionless time t_{Dj} is:

$$t_{Dj} = \frac{Kt}{\phi_j \mu_j C_{tj} L^2} \tag{3}$$

Dimensionless coordinate xDj, yDj, zDj is:

$$x_{Dj} = \frac{x_j}{L}\sqrt{\frac{K}{Kx_j}};\ y_{Dj} = \frac{y_j}{L}\sqrt{\frac{K}{Ky_j}};\ z_{Dj} = \frac{z_j}{L}\sqrt{\frac{K}{Kz_j}} \tag{4}$$

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Put equations (2), (3) and (4) into (1) to obtain the dimensionless reservoir flow equation as follows:

$$\frac{\partial^2 \varphi_{Dj}}{\partial x_{Dj}^2} + \frac{\partial^2 \varphi_{Dj}}{\partial y_{Dj}^2} + \frac{\partial^2 \varphi_{Dj}}{\partial z_{Dj}^2} = \frac{\partial \varphi_{Dj}}{\partial t_{Dj}} \tag{5}$$

Equation (5) shows that all the anisotropic problems can be converted into the isotropic problems.

The initial condition of the reservoir is shown as follows:

$$\varphi_{Dj} = \varphi_{Di} \tag{6}$$

The border condition of the reservoir:

$$\frac{\partial \varphi_{Dj}}{\partial x_{Dj}} + \frac{\partial \varphi_{Dj}}{\partial y_{Dj}} + \frac{\partial \varphi_{Dj}}{\partial z_{Dj}} = 0 \tag{7}$$

Equation (6) and (7) can be solved by the source function generated by the Green's function.

2.2. The Flow Equation of Wellbore

When the fluid flows in the wellbore, the flow pressure gradient can be expressed as:

$$\frac{dP}{dl} = -\frac{4\tau_w}{D} - \rho\frac{g}{g_c}\sin\theta - \frac{z}{g_c A}\rho V q_1 \tag{8}$$

In equation (8) τ_w is the friction stress of wellbore:

$$\tau_w = \frac{1}{7g_c} f \rho V \left|V\right| \tag{9}$$

f is the coefficient of wall friction. For the production well:

$$f = \frac{16}{N_{Re}}\left(1 + 0.04304 N_{Rew}^{0.6142}\right) \tag{10}$$

For the injection well:

$$f = \frac{16}{N_{Re}}\left(1 - 0.0625\frac{\left(-N_{Rew}\right)^{1.3056}}{\left(N_{Rew} + 4.626\right)^{0.2724}}\right) \tag{11}$$

Here, N_{Re} refers to the wellbore flow Reynolds number, N_{Rew} refers to the wall flow Reynolds number. For the producing well, $N_{Rew} > 0$; for the injection well, $N_{Rew} < 0$;

3. PRODUCTIVITY EQUATION SOLUTION

We solved this simultaneous mode by the numerical algorithm of Laplace changing proposed by Zhao and Laplace [6], the numerical algorithm of Laplace inversion proposed by Stehfest as well as the fast linear algebra algorithm of the conjugate gradient method [7, 8], and applied it to the calculation of pressure and production of multi-layer reservoir with block.

Taking the well which passes through two disconnected oil layers as an example, we established a mathematical mode coupling the wellbore and reservoir and solve this model, thus obtaining the solution to the production and pressure of the oil well. For the multi-wells with multi-layers, the situation is similar. The model is illustrated as in Fig. (**1**).

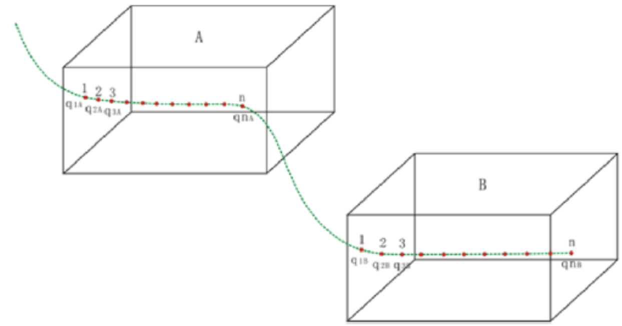

Fig. (1). Model illustration.

This well first goes through block A from the left side and is divided into n horizontal sections, and then goes out of the block A from the right side. Then, it goes downside and through block B, divided into m horizontal sections.

Assume the dimensionless potential energy at position1A is $\varphi_{D1A}(t)$, the total dimensionless production of the oil well is Q_D. At any moment t, the equation can be listed as follows:

$$\sum_{i=1}^{n} q_{DiA}(t) + \sum_{j=1}^{m} q_{DjB}(t) = Q_D \tag{12}$$

$$\begin{aligned}
&\varphi_{D1A} = \sum_{i=1}^{n}\int_0^t q_{DiA}(\tau)S_{1iA}(t-\tau)d\tau + \sum_{j=1}^{m}\int_0^t q_{DjB}(\tau)S_{1jB}(t-\tau)d\tau \\
&\varphi_{D2A}(t) = \varphi_{D1A}(t) + \Delta\varphi_{Df12}(t) + \Delta\varphi_{Da12}(t) \\
&= \sum_{i=1}^{n}\int_0^t q_{DiA}(\tau)S_{2iA}(t-\tau)d\tau + \sum_{j=1}^{m}\int_0^t q_{DjB}(\tau)S_{2jB}(t-\tau)d\tau \\
&\cdots\cdots \\
&\varphi_{D(m+n)A}(t) = \varphi_{D1A}(t) + \Delta\varphi_{Df1(m+n)}(t) + \Delta\varphi_{Da1(m+n)}(t) \\
&= \sum_{i=1}^{n}\int_0^t q_{DiA}(\tau)S_{(m+n)iA}(t-\tau)d\tau + \sum_{j=1}^{m}\int_0^t q_{DjB}(\tau)S_{(m+n)jB}(t-\tau)d\tau
\end{aligned} \tag{13}$$

Equations (12) and (13) contain a total of m + n +1 equations which could be used to solve m + n +1 unknowns. Assume that QD is known, there are (m + n +1) unknowns, $\varphi_{D1A}, q_{1A}, q_{2A}, \ldots\ldots q_{nA}, q_{1B}, q_{2B}, \ldots\ldots q_{mB}$. All the source functions in the equations could be solved by the Newman multiplication principle. However, as all the variation items of dimensionless potential energy are the functions of production, it is necessary to use the iterative method to solve (12) and (13). All the variation items of potential could be determined by the following formula:

$$\begin{cases}
\Delta\varphi_{Df12}(t) = -\dfrac{1}{\varphi_i}\left(\dfrac{4}{D}\Delta L_{12}\tau_{w12}\right) \\
\tau_{w12} = \dfrac{1}{2} f \rho V_{12}^2 \\
\Delta\varphi_{Da12}(t) = -\dfrac{1}{\varphi_i}\left(\dfrac{2}{A}\rho V_{12} q_{12}\right)
\end{cases} \tag{14}$$

In this equation, $\Delta\varphi_{Df12}(t)$ refers to the potential difference between points 1A and 2A caused by the wellbore friction. $\Delta\varphi_{Da12}(t)$ represents the potential difference between points 1A and 2A at any time. ΔL_{12} means the length between points 1A and 2A. V_{12} is the axial flow velocity between points 1A and 2A. q_{12} is the average flow of the well wall between points 1A and 2A. f is determined by (10) and (11). The potential difference of other well sections can be obtained by the similar method.

Laplace transform is conducted for equations (12) and (13) and then it is solved directly. It can be obtained by the convolution nature of the Laplace transform that:

$$\sum_{i=1}^{n}\bar{q}_{DiA}(s)+\sum_{j=1}^{m}\bar{q}_{DjB}(s)=\frac{Q_D}{s} \tag{15}$$

Obtain:

$$\begin{aligned}
&\bar{\varphi}_{D1A}(s)=\sum_{i=1}^{n}\bar{q}_{DiA}(s)\bar{S}_{1iA}(s)+\sum_{j=1}^{m}\bar{q}_{DjB}(s)\bar{S}_{1jB}(s)\\
&\bar{\varphi}_{D2A}(s)=\bar{\varphi}_{D1A}(s)+\Delta\bar{\varphi}_{Df12}(s)+\Delta\bar{\varphi}_{Da12}(s)\\
&\qquad=\sum_{i=1}^{n}\bar{q}_{DiA}(s)\bar{S}_{2iA}(s)+\sum_{j=1}^{m}\bar{q}_{DjB}(s)\bar{S}_{2jB}(s)\\
&\bar{\varphi}_{D(m+n)A}(s)=\bar{\varphi}_{D1A}(s)+\Delta\bar{\varphi}_{Df1(m+n)}(s)+\Delta\bar{\varphi}_{Da1(m+n)}(s)\\
&\qquad=\sum_{i=1}^{n}\bar{q}_{DiA}(s)\bar{S}_{(m+n)iA}(s)+\sum_{j=1}^{m}\bar{q}_{DjB}(s)\bar{S}_{(m+n)jB}(s)
\end{aligned} \tag{16}$$

Equations (14) and (15) correspond to the equation set of (12) and (13) in Laplace space, and it can be solved by an iterative method, and then be transformed to real space by the Stehfest algorithm.

4. MODEL VALIDATION

To verify the effectiveness of the model, calculate the pressure drop and pressure drop derivative of horizontal wells in sand by this model and then compare the results with that of the saphir4.02 software.

The parameters of the validated model are: the length of the sand body xe = 2.3km, the width of sand body ye = 2.0km, the height of sand body ze = 10m, the flow inside diameter 0.1m, formation permeability k = 40mD, formation porosity 0.2, strata compression factor 5×10^{-10}/Pa, oil viscosity 1mPa.s, oil of Japan 100 square / day. Horizontal well position: x = 1-1.3km, y = 1km, z = 5m.

As can be seen from Fig. (**2**), the calculation results of this model are more consistent with that of the commercial software, so the results of this model are very reliable.

5. CASE STUDY

Apply this model to a multi-layer reservoir with blocks which has two sand bodies in the vertical direction, as shown in Fig. (**3**). First, simplify them by their geometry and dimensions and then simplify the "upper 46" on the top part sand body into two parallel rectangular ones, and simply the "lower 48" at the bottom into a rectangular one. The shape and reservoir parameters of the simplified sand body are as shown in Fig. (**3**) and Table **1**.

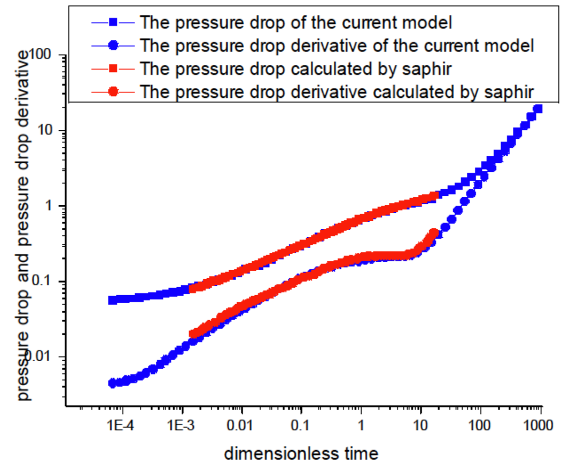

Fig. (2). Comparison of the current Model and commercial software Saphir.

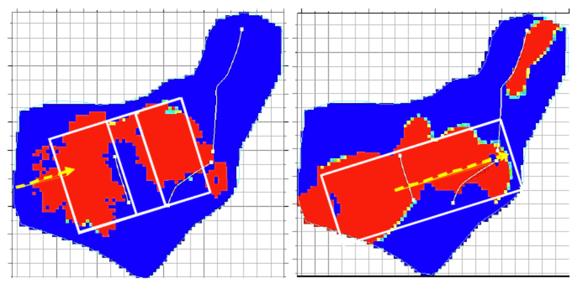

Fig. (3). Sand bodies and their simplified diagram (left "upper 46", right "upper 48", the dotted line represents the well trajectory).

Table 1. Table of reservoir parameters.

Sand Layer	S(km^2)	h(m)	Φ	K(mD)	μ(mPa.s)	L(m)	W(m)
upper 46	2.60	18.46	0.18750	74	2	2000	1300
upper 48	3.12	12.50	0.20512	92	2	3180	981

5.1. Productivity Distribution

Assume that a horizontal well penetrates through the sand body in the left portion of "upper 46" from the middle in the left end to the right end, it continues to pierce downward into the "upper 48" and through from the right end. Based on the productivity model proposed in this paper, the distributions of pressure and yield of different horizontal wells in wellbores are calculated, as shown in Figs. (**4** and **5**).

As can be seen from Fig. (**4**), the pressure drop trends at the bottom can be divided into three stages: radial flow, linear flow and quasi-steady flow. At the stage of radial flow, the pressure drop at the bottom of the well is not so obvious; when the pressure drop pass to the upper and lower boundaries of the formation and goes into the linear flow phase, the pressure drop begins to increase significantly; when the pressure drop passes to the horizontal border of the reservoir and enters the stage of quasi-steady flow, the rate of pressure drop remains constant, but the pressure drop derivative shows linear increase with the increase of time.

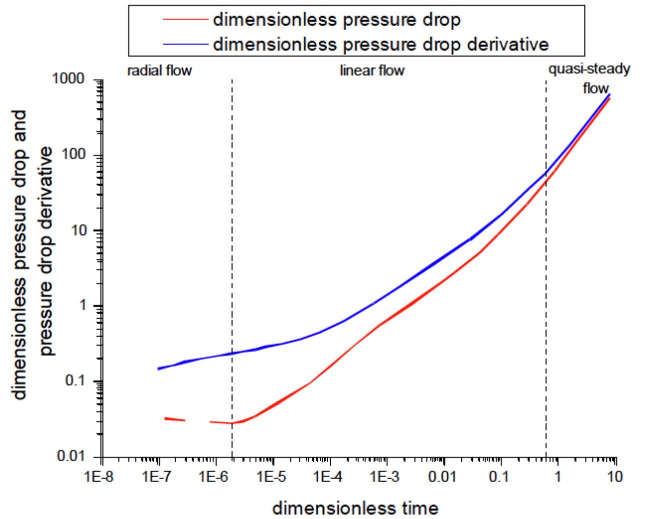

Fig. (4). Pressure drop of the horizontal well and pressure drop derivative.

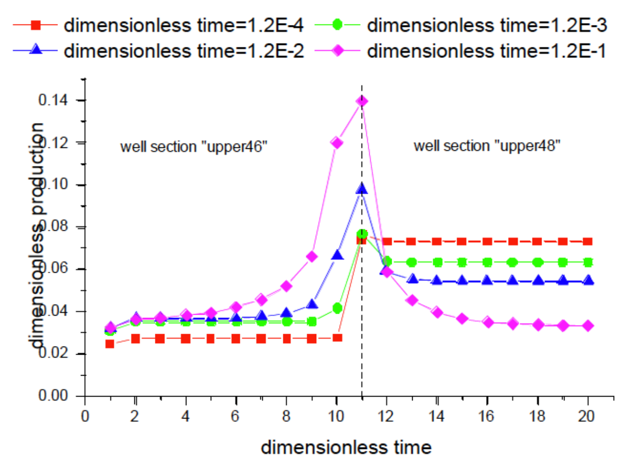

Fig. (5). Variation of production in different sections of the horizontal well.

Since the horizontal sections at the "upper 46" completely penetrated through the left portion of "upper 46", the distributions of productions in each well section remains consistent. However, as the well section piercing "upper 48" does not penetrate from one end of the segment, the first node of the well is high yielding, while the production of other nodes remains uniform. Due to the shorter length of the horizontal section through "upper 46" than through "upper 48", wellbores at the "upper 46" have higher production than "upper 48", but with the passage of time, after reaching the border, the size of the volume of sand body becomes the dominant factor in the production, wellbores at "upper 46" yield slightly higher than "upper 48".

5.2. Production of Different Types of Wells

There are four different ways through two sand bodies for wells (see Fig. (**6**)): the first one is vertical well penetrating through "upper 46" and "upper 48" from the left-center part of " upper 46"; the second is deviated well diagonally piercing through "upper 46" from the edge of the left-center of "upper 46" and coming out from the 1/2 part of the bottom of "upper 48"; the third is deviated well diagonally penetrating through "upper 46" the edge of the left-center of "upper 46" and coming out from the 3/4 part of the bottom of "upper 48"; and the fourth is double-step horizontal well penetrating through the left-half sand body of "upper 46" from its left center and coming downward out from the right-center of "lower 48".

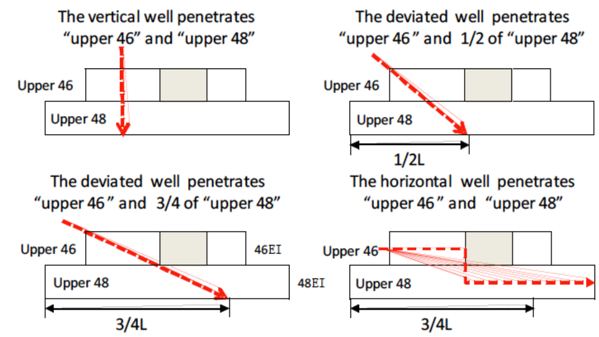

Fig. (6). The cross-sectional view of oil well penetrating the sand body.

Based on the productivity model established in this paper, the dimensionless productivity and dimensionless pressure drop derivative of four well types at different times were calculated, with results of the dimensionless productivity as shown in Fig. (**7**).

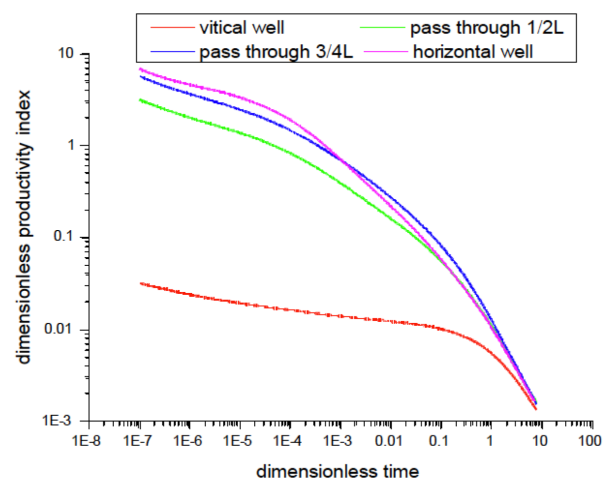

Fig. (7). Exponential curve of the dimensionless productivity.

As can be seen from Fig. (**7**), during the initial production process, the productivity of the vertical well is significantly lower than the deviated and horizontal well before the pressure drop reaches the outer border of the reservoir. Besides, the deviated well penetrating through 1/2 of the reservoir has lower productivity than that through 3/4 of the reservoir, and the horizontal well has the largest productivity at the beginning but later goes between the deviated well through 1/2 and 3/4 of the reservoir. In the late production, the productivity of all wells converges when the pressure drop reaches the outer border of the reservoir.

The results show that if the sand body in the block has small oil-bearing area and good physical properties, the horizontal and deviated wells have higher productivity in the initial stage of radial flow and linear flow, and their productivity is not so superior in the stage of quasi-steady flow. The relative volume, permeability difference of sand bodies and the relative position as well as the length of the horizontal well have a great influence on the feed flow of the horizontal well. Therefore, in mining the reservoir with blocks of the horizontal well, the relationship between injection and production becomes relatively complex, thus increasing the difficulty of reserves control. Conversely, greater oil-bearing area of sand body indicates the time that the pressure drop spreads to the border, and the productivity advantages of the deviated and horizontal wells are more obvious.

CONCLUSION

(1) In the productivity model, the mathematical model of unsteady flow is adopted, which is closer to the actual flow status of reservoir fluid. Thus, it could better describe the flow of reservoir fluids.

(2) We have established a mathematical model of unsteady flow under the condition of coupled reservoir and complex wellbore and taken into consideration the flow friction, momentum changes and other complicating factors in the wellbore flow model.

(3) In this model, each well comprises a plurality of vertical well sections, inclined well sections and the horizontal well sections. Therefore, it allows a well to pass through several disconnected oil layers, which is more in line with the development of the reservoir with blocks.

(4) In the reservoir with multiple sandy bodies and blocks, before the pressure drop spreads to the border of the sand body, the horizontal well productivity> the inclined well productivity > the vertical well productivity; when the pressure drop spreads to the closed border of the sand body, the horizontal well productivity = the inclined well productivity = the vertical well productivity.

(5) The position of the wellbore in the sandy body only determines the time in which the pressure drop reaches the border. Before the pressure drop reaches the border, the position of the wellbore will have an influence on the productivity. The wellbore, located in the center of the reservoir, could better take advantage of the energy of the reservoir to displace the crude oil. Therefore, only when the oil-bearing area of the sand body is large, the position of the wellbore in the sandy body could affect the productivity. Otherwise, it is negligible.

SYMBOL DESCRIPTION

Φ	=	Flow potential，f
x, y, z	=	Coordinates
μ	=	Viscosity，mPa.s
K	=	Permeability，mD
B	=	Volume factor，f
P	=	Formation pressure，MPa
L	=	Reference length，m
q_1	=	The inflow or outflow volume at the unit length of the wellbore，m^3/d
V	=	The axial velocity of the corresponding well section; for the production well, q1 and V are positive values; for the injection well, q1 and V are negative values，m/s
D	=	Diameter of the wellbore，m
g	=	Gravitational acceleration，m/s^2
g_c	=	Conversion coefficient of gravitational acceleration
ρ	=	Wellbore fluid density，Kg/m^3
t	=	time, d
C	=	Compressibility，1/MPa
ϕ	=	Porosity, f
θ	=	The angle between the wellbore and horizontal direction, °
τ_w	=	Wellbore friction stress, MPa
f	=	Friction coefficient of the wall, f
N_{Re}	=	Reynolds number
Subscript j	=	The number of layers
Subscript D	=	Dimensionless quantity
Subscript A、B	=	Symbol of sand body
Subscript m、n	=	Location in the horizontal well section

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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Declared none.

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